

Regular Paper

## Simulation of a Convective Flow Forming a Pattern of the Human Brain

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**Abstract**: To explain consistently determining the global shape of the brain, the vortex model of the brain was proposed by Nakada. He explained that structural organization of the brain is guided by the self-organization pattern based on the principal rule of free thermal convection. In the present study, a computer simulation of a simple thermal-driven convective-diffusive flow was performed to clarify the fundamental relation between this type of the flow and formation of the brain shape. The incompressible Navier-Stokes equations containing the external force dependent on temperature, and an equation for temperature were solved with projection method. The multi-directional finite-difference method was employed to discretize the governing equations. Computational results were suitably visualized and found to be consistent with the arguments in the vortex theory that the thermal-driven convective-diffusive flow is an essential mechanism for determining the global shape of the human brain.

**Keywords**: Simulation, Convective diffusive flow, Brain, Visualization.

### 1. Introduction

With rare exception, genes represent the principal blueprint of all biological organisms, coding for both structural and functional proteins. However, it is obviously not applicable to the structural complexity of the human brain. The human brain contains more than one hundred billion neurons and  $10^{14}$  synapses. It can be easily deduced that a deterministic blueprint for connectivity of such an enormous number of networks is unrealistic. The most immediately obvious concept is a self-organizing neural network. However, principles of self-organizing neural networks are not sufficient for defining the global shape of the brain.

In the present study, a hypothesis that provides a consistent explanation for determining the global shape of the brain was proposed by Nakada (2003). In his theory, the vortex model of the brain was discussed, namely, structural organization of the brain is guided by the self-organization pattern based on the principal rule of free thermal convection.

In this study, a thermal-driven convective-diffusive flow was simulated under a condition suggested by the vortex model of the brain. Computational results were suitably visualized using computer graphics technique.

### 2. Vortex Model of the Brain

The ontogenic process of cortical organization and, hence, the process of shaping the brain is phenomenologically well described (Fig. 1). The cells that eventually form the six-layered cortex all originate in the germinal matrix (GM) adjacent to the lateral ventricle. Each cell (neuroblast)

migrates to the surface of the brain along physical scaffolding created by radial glial fibers (RGF). Neuroblasts differentiate as they reach the cortical plate, thereby forming the cortex. Hence, the outline of the global structure of the brain is provided by radial glial fibers. This implies that the rule of determining how radial glial fibers advance is the rule that defines the global shape of the brain.

In the vortex model of the brain, it is shown that radial glial fibers follow the basic principles of convection. Mammalian physiology is characterized by a steady core body temperature. Because heat generated by metabolic activities sustaining body temperature is eventually released into the environment, there is continuous heat dissipation from the core of the body to the atmosphere. On the other hand, from the ontogenic standpoint, it is unlikely that gravity plays a significant role in determining the individual shape of the brain since the head-gravitational field relationship of the freely moving fetus is arbitrary. Rather than gravity, the main driving force in creating a convection pattern in biology is likely to be heat dissipation into the surroundings analogous to the Marangoni variant.

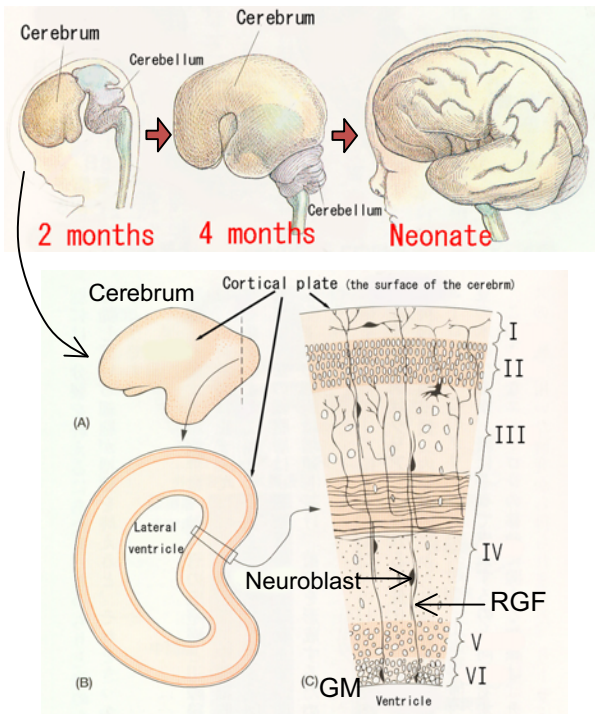


Fig. 1. Six-layered cortex in the brain (Bloom et al., 2001).

### 3. Objective

Nakada simulated a convective flow. His computational result suggested a relevance of the principal rule of a convective flow to defining the shape of the brain. In his computation, a thermal convective flow was solved under the gravitational field. It is essential that a pattern shown in the brain is formed with a mechanism of a convective and diffusive flow, rather than what is the driving force in convection. However, in the computational result, because gravity played an essential role to determine the flow schema, it is not easy to understand the correspondence between mechanisms of forming the brain and convective flow. Therefore, in the present study, a 3-d flow simulation is carried out with a simple convective and diffusive flow model without the driving force in a specific direction such as gravity.

## 4. Computational Method

### 4.1 Governing Equations

A thermal-driven convective-diffusive flow was simulated by solving the compressible Navier-Stokes equations under almost incompressible conditions in which the density term was replaced with a value that depended only on temperature difference (Kuwahara, 1984; Komurasaki et al., 2002). The governing equations were as follows:

$$\operatorname{div} \mathbf{u} = \alpha \operatorname{div} \left( \frac{1}{\operatorname{Re} \cdot \operatorname{Pr}} \operatorname{grad} T \right) \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \operatorname{grad}) \mathbf{u} = -\alpha T \operatorname{grad} p + \alpha T \operatorname{div} \left( \frac{1}{\operatorname{Re}} \operatorname{grad} \mathbf{u} \right) + \mathbf{K} \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \operatorname{grad}) T = \alpha T \operatorname{div} \left( \frac{1}{\operatorname{Re} \cdot \operatorname{Pr}} \operatorname{grad} T \right) \quad (3)$$

$$\mathbf{K} = -\frac{c}{|\text{grad}T|} \frac{\Delta T}{T_b} \text{grad}T \quad (4)$$

where  $\mathbf{u}$ ,  $T$  and  $p$  represented the velocity vector, temperature and pressure respectively. Density  $\rho$  was given by  $\rho = 1/(\alpha T)$  with constant  $\alpha$ . Re and Pr were Reynolds and Prandtl numbers respectively. The vector  $\mathbf{K}$  was the driving force in convection.  $c$  was an arbitrary positive constant and  $\Delta T$  indicated temperature difference from the base temperature  $T_b$ . In the real fetus brain, the main driving force of convection has not been clarified, except that it is suggested to be a force analogous to the Marangoni variants. Our simulation aimed at investigating whether the simple thermal-driven convective-diffusive flow model given above can reproduce the structural patterns of the human brain, rather than validating the nature of the driving force.

To solve the governing equations, a multi-directional finite-difference method (Kuwahara, 1999) was employed to discretize the governing equations. They were solved using the projection method (Chorin, 1968; Takami and Kuwahara, 1974). Hence, a pressure field was obtained by solving the Poisson's equation. Space derivatives were discretized using a central difference approximation with exception for the convective terms. For the convective terms, a third-order upwind scheme (Kawamura and Kuwahara, 1984) was used to stabilize the computation. It has been found to be the most suitable for high-Reynolds number flow computations. The second-order Crank-Nicolson implicit scheme was used for time integration. The equations were iteratively solved at each time step by SOR method. A multi-grid method was utilized to solve the Poisson's equation.

#### 4.2 Computational Conditions

The convective flow simulation was performed in a 3-d computational domain (Fig. 2). The shape of the boundary was an elliptic sphere with two other small elliptic spheres attached to the lower part of it and an axial partitioning wall placed in the upper part of it. This three dimensional model was a representation of the genetical blueprint.

The initial temperature was set to the highest value of 1 °C in a spherical region at the center of the domain and the lowest value of 0 °C within the partitioning wall and the outside boundary. The base temperature  $T_b$  was equivalent to the lowest temperature.

In this computation, uniform grid in Cartesian coordinate system was utilized for the cube-type whole domain. Grid points were  $65 \times 65 \times 65$ . The constant parameters were as follows: Re=100,000, Pr=1.0,  $\alpha=1/273$  and  $c=100$ . Non-slip and constant temperature conditions were applied on the boundary.

## 5. Computational Results

### 5.1 Comparison of a Simulated Temperature Field with the Brain Shape

The computation was carried out on a single CPU (Intel Pentium 4). Computational results were visualized suitably by using a software package *Clef3D* developed at the Institute of Computational Fluid Dynamics.

Figure 3 shows a coronal cross section of the human brain (a) and an instantaneous temperature field in the plane C shown in Figs. 5(b) and (c). The temperature shading was visualized in two different ways: colored from black to white for the range from 0 °C to 0.7 °C (b) and colored from blue to red for the same range with contour lines added (c).

The temperature distribution well reproduced the global structure of the cortex as characterized by "grooves." Note that the clear difference in the structure near and within the center region of the brain is not significant to the present study because this region is dominated by the lateral ventricle and should be formed by other mechanisms than the convective-diffusive flow.

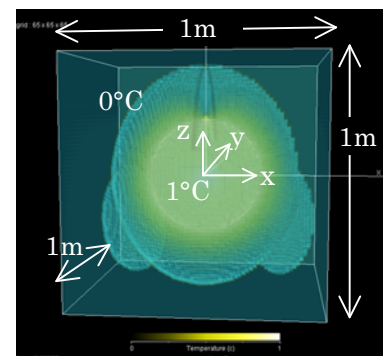


Fig. 2. Computational domain.

Figure 4 shows only an essentially significant part of the flow structure indicated in Fig. 3.

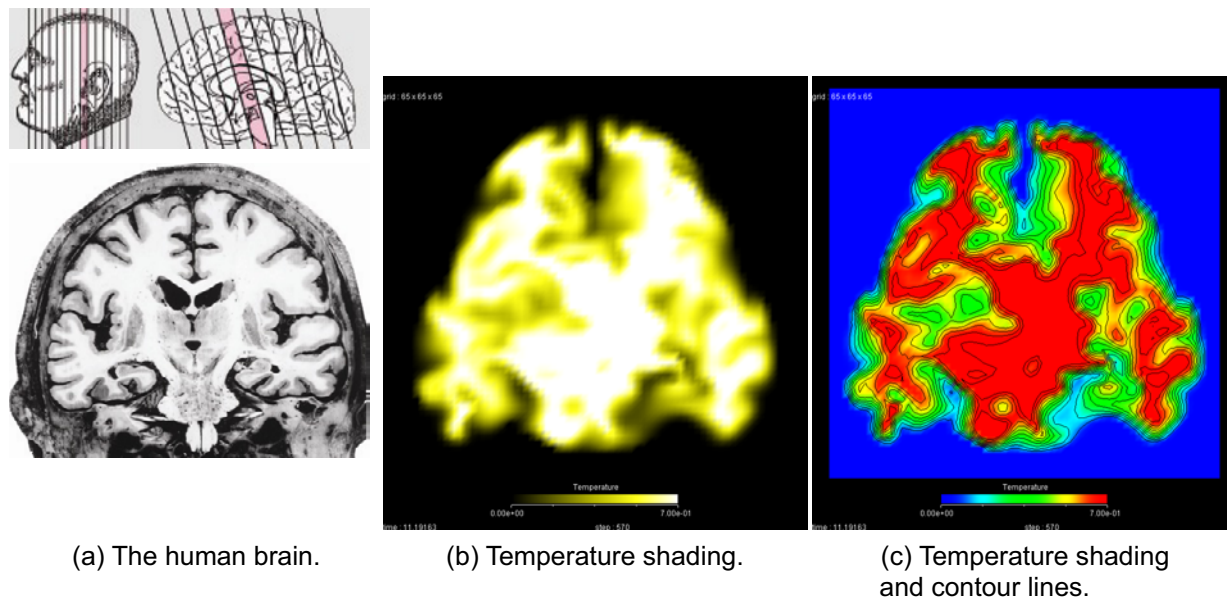


Fig. 3. The human brain and a temperature field in a coronal cross section.

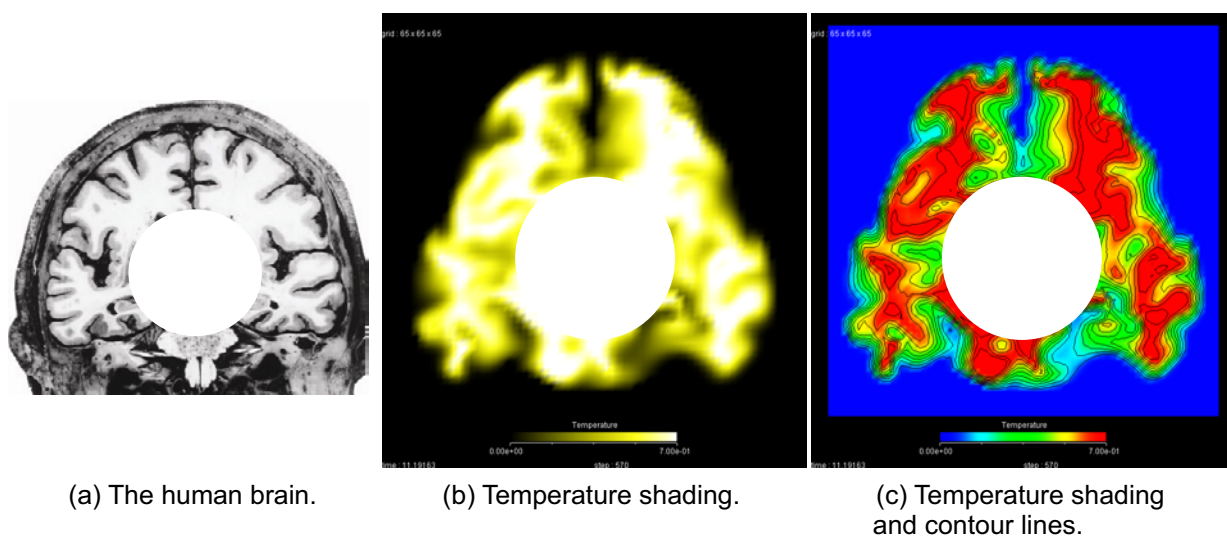


Fig. 4. The human brain and a temperature field in a coronal cross section with screening the center region.

### 5.2 Time Development of the Flow Field

Because of the dynamic nature of convective flows, time development of the temperature field is of another interest. A time sequence of the temperature shading was presented in Figs. 6 to 8 for the planes A to C (indicated in Fig. 5), respectively. The planes A and B corresponded to horizontal sections of different locations, while the plane C was selected as a representative coronal section of the brain. Figure 9 shows the same time sequence by a contour surface of the 3-dimensional temperature distribution. In Fig. 10, time development of the flow field was presented with streamlines starting from the domain center.

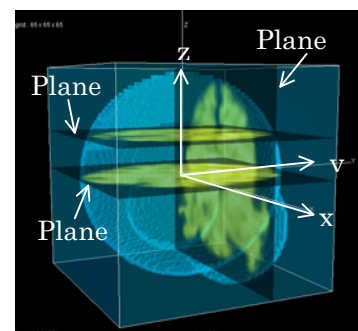


Fig. 5. Temperature shading in three planes.

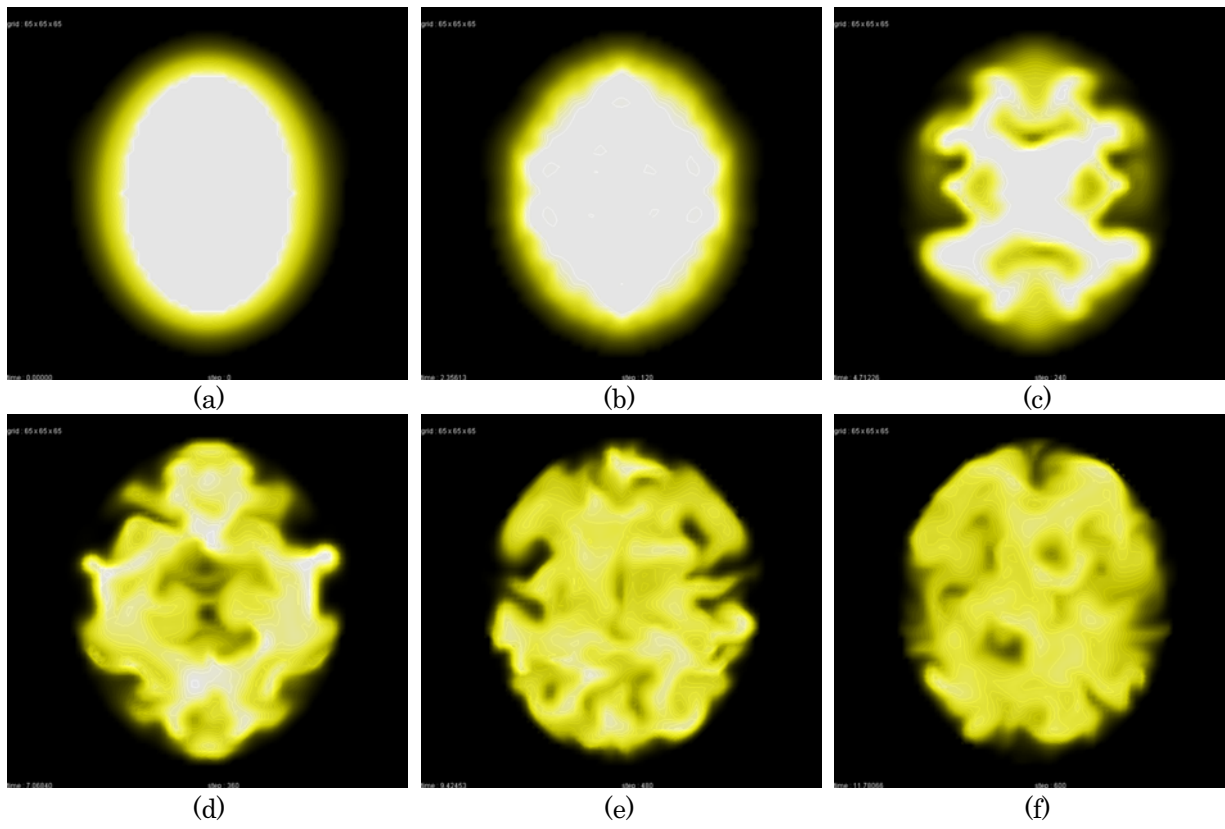


Fig. 6. Time development of the temperature field in plane A.

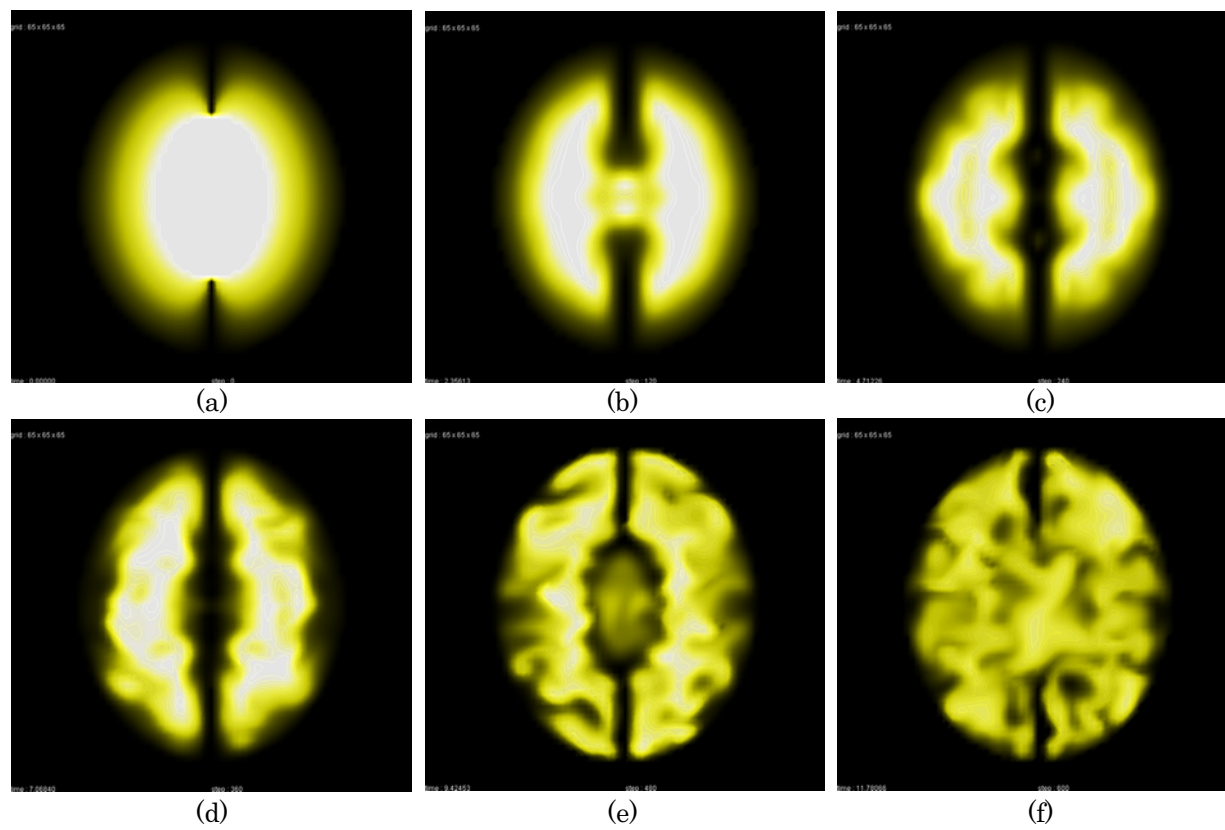


Fig. 7. Time development of the temperature field in plane B.

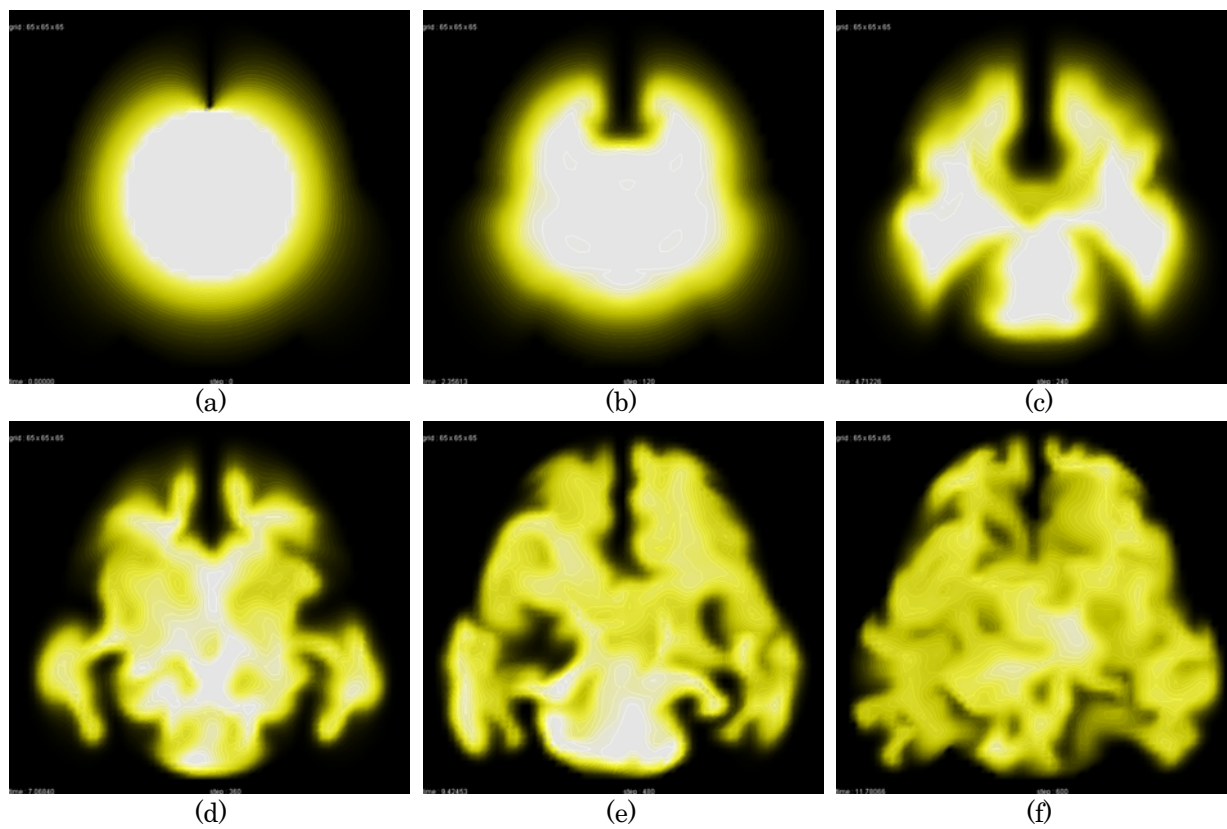


Fig. 8. Time development of the temperature field in plane C.

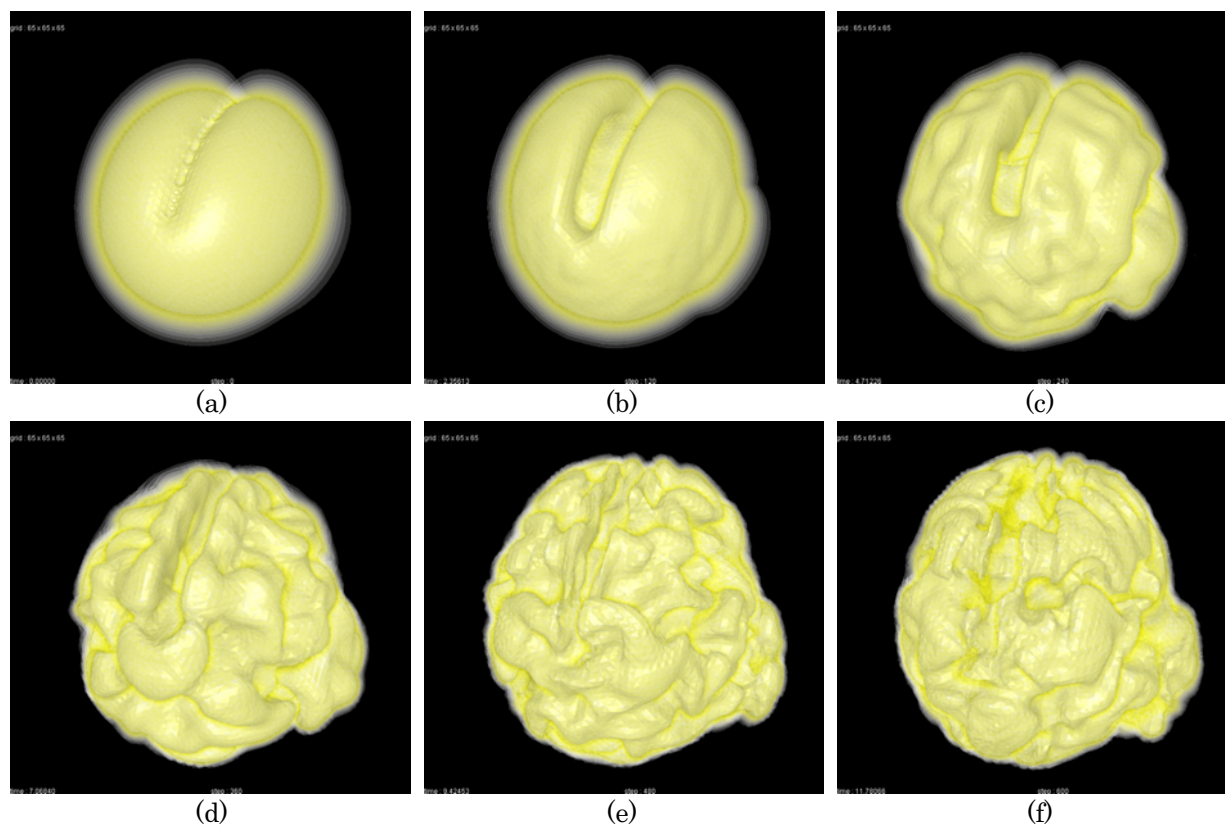


Fig. 9. Time development of temperature contour surfaces between 0°C and 0.7°C.

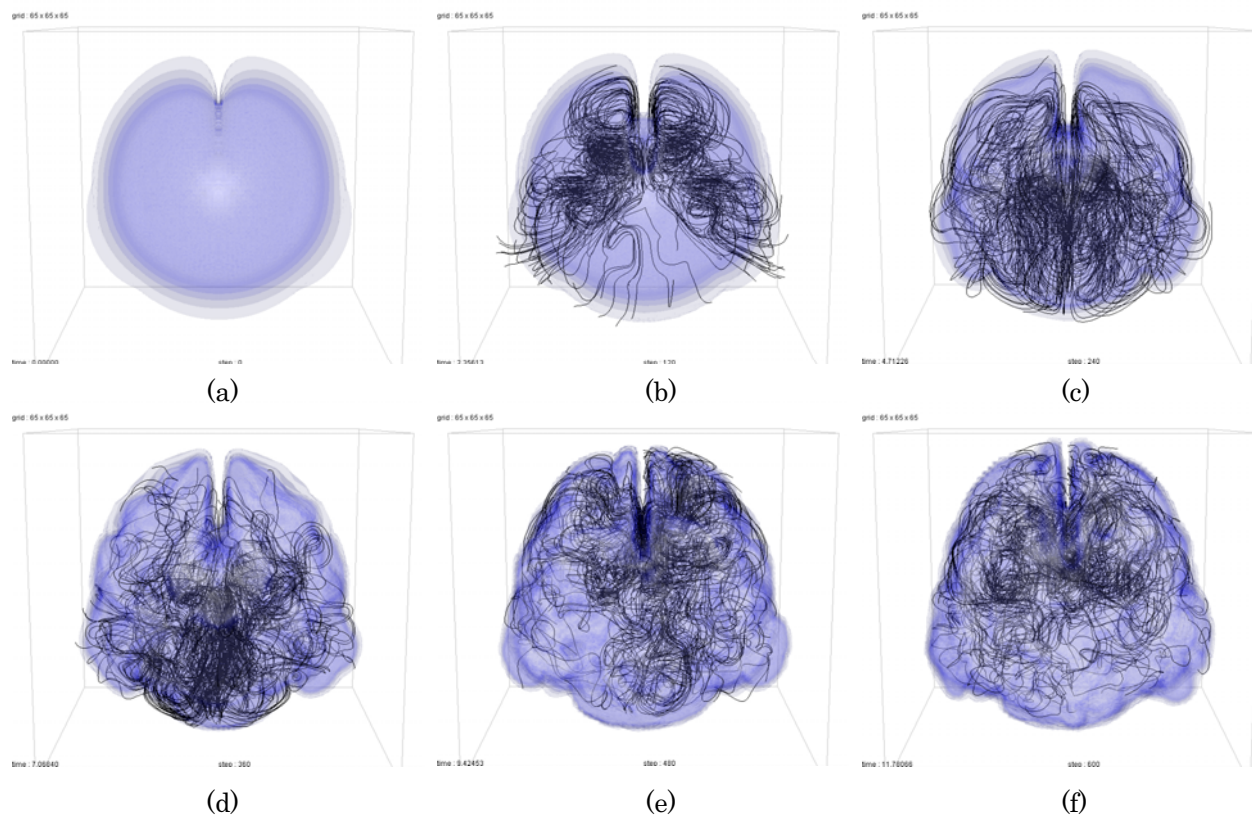


Fig. 10. Time development of the flow field expressed by streamlines starting from the domain center.

## 6. Conclusion

Our simulation using the simple flow model has suggested that the principal rules of thermal-driven convective-diffusive flow take an essential role in forming the cortex structure. The present results are consistent with the arguments in the vortex theory of the brain.

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